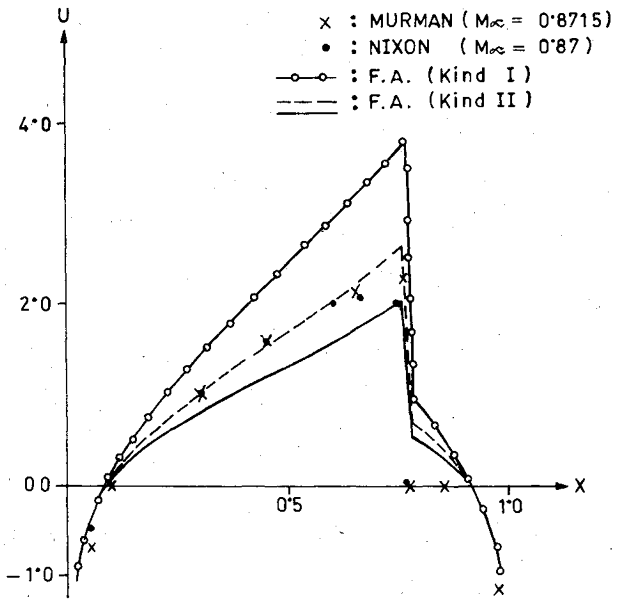

Fig. 3 Fuzzy estimation of velocity distribution for a parabolic arc profile.

similar to that with shock would be obtained only if $(U_p)_{max} \geq 1$. In Fig. 1, results for NACA 0012 profile for different freestream Mach numbers have been plotted. A continuous solution has been obtained up to $M_\infty = 0.75$. After that a discontinuous solution appears. Agreement with the shock position is excellent, as is shown in Figs. 2 and 3. Disagreement is mainly in shock strength in both the cases.

Fuzzy Algorithm (Type II)

For the modification of the result obtained by type I, a fuzzy generation algorithm,[1] or fuzzy algorithm of type II, is applied. This algorithm serves to generate different consistent patterns. Following this algorithm a set of feasible patterns will be obtained and then from these feasible patterns, the most preferable one will be selected.[4] But, as we shall see in the following discussion, it is better to choose a set of preferable patterns than the conventional notion of choosing the most preferable one.[5]

It was observed in Ref. 2 that the optimal value of a in Eq. (2b) is 1 for shock-free supercritical flow. But for increasing Mach number and for flow with shock, the value of $U(X,Y)$ at the point of $(U_p)_{max}$ is found to be a good fit for decreasing a in Eq. (2b). Moreover, it appears from Eq. (2) that $I(X,Y)$ will be smaller for the value of a nearly ½. Thus, our choice of a is

$$a = \tfrac{1}{2} + \epsilon \tag{5}$$

where ϵ is a small positive real number. Different scale factors could be evaluated using different ϵ corresponding to the point of $(U_p)_{max}$ on the profile axis. A set of results would be generated using these scale factors on the discontinuous solution obtained using type I. In Fig. 3, results have been shown for parabolic arc profile for $M_\infty = 0.87$. The continuous line is for $\epsilon = 0.01$; the dotted line is for $\epsilon = 0.31$. The set of preferable patterns could be defined for ϵ in the interval [0.01, 0.31]. A more restricted choice of ϵ could be possible, at the user's discretion. In the present work ϵ is defined as:

$$\epsilon = \frac{M_\infty}{[(U_p)_{max}]^2} \tag{6}$$

For the particular case in Fig. 3 it becomes 0.1, which is in the defined interval. This definition (6) is used for the NACA 0012 profile for $M_\infty = 0.791$, and results are shown in Fig. 2. The overall agreement is quite satisfactory.

The novel feature of these fuzzy algorithms is the huge time reduction. CPU time for the computation of the result shown in Fig. 2 or 3 is approximately 3 s on the B 6700. We are now trying to extend this method to three-dimensional cases and to generalize this approach for various applications.

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Axial Wavenumber Measurements in Axisymmetric Jets

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Introduction

THE existence of orderly structures in axisymmetric jets of air has been well established since the first observations by Mollo-Christensen.[1] Since that time, many investigators[2-12] have studied the coherent structures and their importance in the generation of noise.

Kibens[7] showed that, for incompressible flows ($M<0.3$) with a laminar shear layer at the exit of the jet, an instability was generated whose wavelength scaled on the thickness of the shear layer. As the flow progressed downstream, the shear layer became thicker and the frequency of the coherent structure halved according to a vortex pairing phenomenon, such that the wavelength of the new spectral component was again scaled upon the thickness of the shear layer. Eventually, after several pairings during the first few diameters of the flow, a coherent structure whose wavelength scaled on the diameter of the jet was obtained and the vortex pairing ceased. The resulting Strouhal number (fD/Uo) was around 0.4.

This behavior is very different from the compressible jets ($M \geq 1.4$) studied by Morrison and McLaughlin.[9] The supersonic jets studied possessed laminar shear layers at the exit of the nozzle. However, the initial instability observed in the compressible jets had wavelengths which scaled upon the diameter of the jet. In addition, there was no subharmonic production in the supersonic jets and, hence, no direct evidence of vortex pairing.

Kibens also observed that, if the shear layer was initially turbulent in the incompressible jet, then the pairing process did not appear to occur. The jet initially developed a coherent

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structure whose wavelength scaled on the jet diameter with the same Strouhal number as was eventually reached in the case of the initially laminar shear layer.

The investigations of Kibens and Morrison and McLaughlin indicate that there is a change in the fundamental nature of the coherent structures, depending upon the compressibility of the flow. In addition, Kibens found that the initial development of the coherent structure depended upon the condition of the shear layer at the exit of the jet for the incompressible jet.

Facility

In the present study, three Mach numbers (0.6, 0.7, and 0.8) were considered. The experiments were performed in a free-jet test facility consisting of an axisymmetric nozzle with an exit diameter (D) of 12.5 mm and a contour described by a third order polynomial. Compressed air was supplied by a reciprocating compressor to a 300 mm diameter stilling chamber by way of a storage tank, water aftercooler, water separator/filters, an electronic pressure regulator, and a muffler. The aftercooler was used to maintain the air's stagnation temperature at 298 K.

For the $M=0.6$, 0.7, and 0.8 jets, the Reynolds numbers based upon exit diameter were 184,000, 221,000, and 262,000, respectively. Custom manufactured hot-wire probes operated by a dual channel TSI 1050 anemometer system were used to measure the flow fluctuations. The hot-wire data were decomposed using the method of Horstman and Rose.[13] Spectra, cross spectra, and cross correlations were measured using either a Nicolet 660A Dual Channel Spectrum Analyzer or a Saicor SAI 42 Correlation and Probability Analyzer.

Experimental Results

The axial wave (krD) and azimuthal mode (n) numbers were determined for each frequency investigated. The axial wavenumber has typically been measured using one of two different techniques. The first technique consists of using two flow sensing probes (hot wires and/or microphones). One probe remains stationary while the other is moved in the axial direction to several different locations. At each location, the relative phase difference between the two signals from the probes is determined at the frequency of interest by the use of cross spectra or cross correlations. From plots of relative phase as a function of axial displacement, the axial wavelength and, hence, axial wavenumber, are determined.

This first technique was used by Armstrong,[2] Morrison and Wattanachayakul,[10] and the current investigation. The latter two investigations used two hot-wire probes positioned at the radial location of maximum flow fluctuation level with an azimuthal separation of 180 deg. Armstrong used two microphones fitted with nosecones placed at $r/D=0.5$, with an azimuthal separation of 90 deg.

The second technique used for measuring coherent structures consists of replacing the stationary probe by some type of artificial excitation device. The purpose of the excitation device is to input a disturbance with known frequency, amplitude, and phase into the jet and observe how the jet responds. The advantage of this technique is that the stationary probe is eliminated and, hence, the disturbances created in the flow by the presence of the probe are also eliminated. The main disadvantage of this technique is that if one wants to determine the structure that is naturally occurring in the jet, great care has to be exercised that the excitation does not appreciably alter the jet from its natural condition. Moore[8] and Morrison[14] have shown that it is possible to change the nature of a jet by using excessive levels of artificial excitation for both subsonic and supersonic jets.

Crow and Champagne,[6] Chan,[4,5] Morrison and McLaughlin,[9] Stromberg et. al.,[11] Troutt and McLaughlin,[12] and Morrison and Wattanachayakul[10] have all used artificial excitation devices in order to measure the axial wavenumber of the coherent structure in subsonic and supersonic jets.

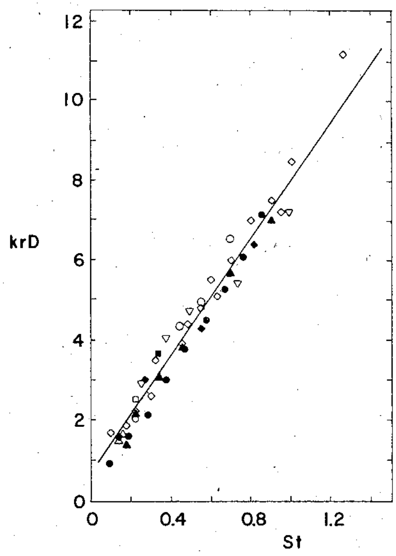

Fig. 1 Axial wavenumber (krD) variation with Strouhal number ($St=fD/U$), ▲ $M=0.30$-0.9[2] ($n=0$ to 3), ◇ $M=0.6$, present study ($n-3$ to 3), ◆ $M=0.7$, present study ($n=-3$ to 3), ▽ $M=0.8$, present study ($n=-3$ to 3), ○ $M=0.9$[11] ($n=0$ and ± 1), ■$M=1.4$[9] ($n=\pm 1$), □ $M=2.1$[9] ($n=\pm 1$), ● $M=2.1$[12] ($n=0$ and ± 1), △ $M=2.5$[9] ($n=\pm 1$).

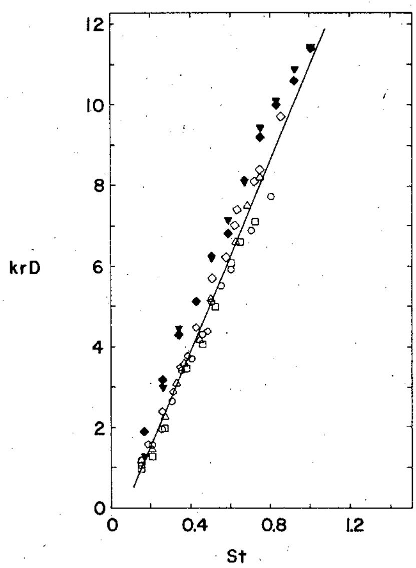

Fig. 2 Axial wavenumber variation with Strouhal number, $M<0.3$, ○ $n=0$[6], △ $n=0$, shear layer,[4] □ $n=0$, jet axis,[4] ◇ $n=0$, acoustic near field,[4] ◆ $n=1$[5], ▼ $n=2$[5].

Crow and Champagne and Chan used acoustical excitation devices in subsonic ($M<0.3$) jets. Morrison and McLaughlin, Stromberg et. al., and Troutt and McLaughlin used a glow discharge device to excite high speed, compressible jets ($0.9 \leq M \leq 2.5$). Morrison and Wattanachayakul used a spark excitation device to excite the flow.

Figure 1 illustrates the axial wavenumber-frequency relationship for both natural and artifically excited jets with Mach numbers greater than 0.3. A linear curve fit of all of the data shows that the wavenumber frequency relationship can be expressed by: $krD=0.7735+7.226St$. It is important to note that these data include jets with Mach numbers ranging from 0.3 to 2.5, Reynolds numbers from 3,700 to over 500,000, initial shear layers that were laminar and turbulent, shear layer thicknesses that ranged from 5 to 16% of the jet exit radius, and various azimuthal modal composition (modes from $n=-3$ to 3). These data show that the axial

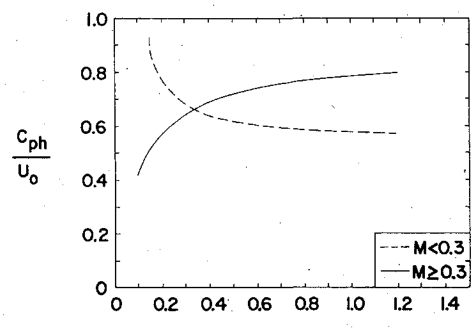

Fig. 3 Nondimensional axial wave speed (Cph/Uo) variation with Strouhal number.

wavenumber-frequency relationship is independent of all of the aforementioned quantities.

Figure 2 contains the remaining wavenumber-frequency data where the Mach number is less than 0.3. The works of Crow and Champagne and Chan compare favorably with a curve fit (for $n=0$) showing that the wavenumber-frequency relationship follows the form: $krD = -0.7435 + 11.714St$. Comparing the slope of this curve fit to the previous one shows that there was a definite difference in the two different Mach number ranges. For both of the incompressible studies, the $n=0$ mode was excited by a speaker in the plenum. For the two higher modes studied by Chan, six acoustic drivers were located around the azimuth of the jet at the exit. The offset of the $n=0$ data from the two higher modes may reflect the different excitation techniques used since, in the compressible data, many different modal compositions all maintained the same axial wavenumber-frequency relationship.

Figure 3 shows the nondimensional axial wave speed (Cph/Uo) for the different frequencies as calculated from the curve fits using $Cph/Uo = 2\pi St/krD$. For the compressible flow data, the wave speed increases with increasing frequency and asympotes to a maximum value of about 87% of the exit velocity of the jet (Uo). For the incompresssible flow, a completely different trend is observed. The speed decreases with increasing frequency and asympotes to a value of about 54% of the exit velocity. In both cases, the wave systems are dispersive, with wave speed varying with frequency.

The data presented show that when the Mach number of an axisymmetric jet is above 0.3, the wavenumber-frequency relationship is independent of Reynolds number, Mach number, initial shear layer condition, and azimuthal modal content. Also, in this Mach number range, the coherent structures move downstream with increasing speed as the frequency increases. For $M<0.3$, the axial wavenumber-frequency relationship is different than for the other jets and exhibits some dependence upon the modal content. These incompressible jets possess coherent structures whose axial wave speed decreases with increasing frequency and is dependent upon the initial shear layer condition. Further investigation is required to determine if this change in the nature of the coherent structure is due to compressibility effects or to other experimental effects such as the level of artificial excitation used.

Acknowledgments

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Computations of Unsteady Transonic Cascade Flows

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Introduction

IN an oscillating cascade there is by definition a fundamental periodicity that occurs every $2N$ blades ($2N$ is the number of blades in the compressor row). The unsteady flow at each blade will have a periodic boundary condition, as in steady flow, but will lag by a phase angle of $(p/N)\pi$ in relation to the neighboring blade, where p is an integer less than or equal to N whose value is determined as part of the flutter solution. In a nonlinear transonic numerical scheme the choice is between computing the entire $2N$ blade sequence with the usual periodic boundary conditions at the extremities or computing a three-blade cascade problem for each specified value of p. These numerical calculations are computationally expensive and it is desirable to reduce the overall cost of a flutter calculation. Both of these choices involve a large amount of computer time for practical cases and, in the case of the first choice, a major development of a computer code. However, it is possible to devise a simpler approach to the problem.

The basic idea of this Note is to devise an elementary problem in which only one blade is in motion, the others being fixed; the motion may be any general time-dependent function. This removes the problem of computing the flow for each blade phase angle. This elementary problem is solved for a particular moving blade and the functional form of the velocity potential for both space and time is then known. Because of periodicity in both space and time, these

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